IC-NOCMAT 2007 Maceió

International Conference on Non-Conventional Materials and Technologies: Ecological Materials and Technologies for Sustainable Building

Maceió, Alagoas, Brazil, 14th-17th October 2007

In Honour of Professor R.N.Swamy

BEHAVIOUR OF BAMBOO *DENDROCALAMUS GIGANTEUS* SUBJECTED TO AXIAL COMPRESSED LOAD

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Abstract: In this paper a device developed to register the geometrical mapping of bamboo to establish the initial imperfections along a two meter long bamboo columns of the species *Dendrocalamus giganteus* is presented. The buckling test results of these segments are presented through the Southwell Diagram, presenting global behaviour of the column with hinges in both sides. These investigations are part of a series of research programs which have been realized to establish the physical and mechanical properties of different bamboo species and development of connections for plane and space trusses. Of great importance for the study of the stability of columns is the evaluation of the imperfection of the elements' geometry, mainly responsible for the loss of loading capacity in relation to the higher limit, given by the theoretical equation of Euler, which is applied to a column. Bamboo, being a natural product, presents deviations in the column axis with immediate consequences on the loading capacity under compression. The regularity and consistency of the obtained results showed that the dimensioning of long bamboo elements is feasible considering the geometrical variation of the bamboo beside other parameters which influence the mechanical strength of this natural material. Finally, recommendations are given for the design of bamboo subjected to compression load.

Keywords: *bamboo, geometrical imperfections, buckling tests.*

Resumo: Neste artigo é apresentado um aparelho desenvolvido para descrever o mapeamento geométrico do bambu com a finalidade de estabelecer as imperfeições iniciais ao longo de elementos de 2 metros de comprimento, da espécie Dendrocalamus giganteus, a serem utilizados como colunas. Os resultados dos testes de flambagem desses elementos são apresentados através do Diagrama de Southwell, que controla o comportamento global de colunas birotuladas. Estas investigações são parte de uma série de programas de pesquisa realizados para estabelecer as propriedades físicas e mecânicas de diferentes espécies de bambu e para o desenvolvimento de conexões para treliças planas e espaciais. As imperfeições da geometria dos elementos são de grande importância para o estudo da estabilidade de colunas, sendo as principais responsáveis pela perda de capacidade de carga em relação ao limite superior, dado pela equação de Euler. Como o bambu é um produto natural, seu eixo apresenta desvios que têm conseqüência imediata sobre a capacidade de carga sob compressão. A regularidade e consistência dos resultados obtidos mostraram que o dimensionamento de elementos longos é controlável considerando as variações geométricas do bambu conjuntamente com outros parâmetros que influenciam a resistência mecânica desse material natural. Finalmente são fornecidas recomendações de projeto para bambus sujeitos a cargas de compressão.

Palavras-chave: *bambu, imperfeições geométricas, testes de flambagem.*

INTRODUCTION

From the structural mechanics point of view, bamboo, mainly in order to counteract wind load and the own weight, acquired several natural geometries which turns it a most optimum structure to the requirement of deflection-compression: - a conical form along the culm, an approximately circular transversal section, a hollow form in most species, which reduces its weight, a gradient rigidity to deflection in the radial direction of the bamboo's cross-section, among others.

The appropriation by man, with the aim of utilization in construction, demands, in the same way as in the biological self-evolution, denominated as " autopoiese" by Maturana [1], that it becomes known to extract the utmost mechanical possibilities of this system already determined, in a way as to establish, with greater precision and safety, the limits of resistance and utilization of the bamboo structure for each type or requirement.

The leap the bamboo structure is thought to have done in the last decades consisted of passing from an intuitive structural application to a controlled one. As well, an attempt was made to determine to which point it becomes viable to control the structural behaviour of bamboo in its natural state, conforming to the type of requirement and environmental conditions. It is a fact that the intuitive application of bamboo in construction, during millenniums, was and still is extremely useful and without great risks for the users within the limits already established. This attributes to the mechanical control, the task to make it possible to establish a safety index for bamboo constructions, making the structure more economical and making some type of requirements feasible, which so far had been avoided, exactly for involving a greater risk and demanding greater knowledge of the mechanical response.

Therefore, we apply to knowledge as a function in order to liberate the evolutional process of humanity and not as a mechanism of restriction and social-political control, in the way that the mechanical control of the traditional structural systems and others ought to facilitate and stimulate the use of this noble plant. The mechanical behaviour control of bamboo has to pass through its geometrical idealization. For all existing natural forms, man created an idealized form, such as straight lines, circles, ellipses, among others, by which all natural forms would be imperfect manifestations, probably a heritage of the thoughts of the Greek philosopher Plato. As a fact, the so-called perfect forms have their advantages as they facilitate industrialization, the execution and mathematical analysis, but their extreme habitual use could become inconvenient as they make people develop prejudice towards natural forms as being "imperfect". This dis-naturalization of the world, which turns the natural world unreal and the virtual world real, a characteristic behaviour in the actual level of scientific-technological development, brings serious damage to the ecosystems.

Therefore, to draw on these raw materials, trying to integrate them in a symbolic contemporary universe and establishing their form of utilization is as well returning them to the sovereignty of nature and Earth. Thus, with the objective to be able to utilize bamboo, as it is made by nature, in airy structures, such as space structures, geodesics and cable stayed structures in general, we present part of our studies in this paper, Moreira and Ghavami[2] which are related to the evaluations of natural deviations in the longitudinal axis and their relevance to the load limit, when the closed tubular sections are subjected to axial compression.

MATERIALS AND METHODS

The first researcher, deducing a mathematical expression for the buckling load of a column, was Leonhard Euler in 1744. For such, all the apparent chaotic phenomena, presenting itself with its many real varieties and relations, was reduced to three relevant variables, adjusted by the square of the irrational number π. One variable represents the rigidity of the material constituent of the column, E, the other represents the geometrical response of the transversal section, when turning around an axis, I, and the last variable is the length of column l. The expression deduced by Euler, $F_E = \frac{\pi^2 EI}{l^2}$, establishes the critical load F_E, for a long column, which corresponds a theoretical load in which the ideal column is equal to a straight form and as well equal to a slightly curved form.

However, in order to reach this expression, other assumptions have already been made, such as: - the material is homogeneous, isotropic and obeys the Hooke law; the column is perfectly straight and has a constant transversal section. Also, the column is joined to the neighbouring structural components by means of perfect hinges at the extremity. In the experiments the Euler load only will be effective to a slender column and the longitudinal axis is perfectly straight one. So, the deviation of the longitudinal axis is a indispensable variable in the understanding of real columns Chages [3].

The study of the behaviour of bamboo under compression leaves us then with some doubts:

- how to measure the initial imperfections of the axis δ_0 ?
- The geometrical approximation of the transversal section is a circular ring with a constant section throughout the element leads mathematically the necessary results? In which position of the element is it convenient to assume the transversal section to be constant?
- Does the density gradient of the transversal section, in radial direction and from the inside outwards, seen in Fig. 1, affect the results ?

- How does the failure of the elements occur?

Figure 1: Density gradient

Mapping of Bamboo

To measure the circumference of the bamboo's geometry with precision a special mapping devise was designed, as shown in Fig. 2a.

a) b)

Figure 2: Mapping Device developed to measure the Bamboo's Geometry

The measuring ring consists of an external ring, which is fixed to the aluminum bars and an internal ring, which contains a dial gauge with the exactness 1/1000*,* see Fig. 2b. This internal ring can turn 360^0 in steps of 15^0 . Knowing the distance from the end of the pointer of the dial gauge to the center of the ring, it is possible to reproduce the whole circumference of the bamboo at a determined height. The external circumference of the bamboo is measured at every 10 cm along the axis.

At every 15^0, 24 points were marked on the surface of the bamboo. At each pair of points, a circumferential distance of 90^0 was supposed to belong to one diameter, in a way that the center of this circumference was obtained by the intersection of 2 diameters. After a turn of 15^0, another center could be determined in the same way. The coordinates of these centers were quite close to each other, but did not coincide, because bamboo is not perfectly circular.

It was assumed that the centroid of each circumference was the arithmetic average of the obtained coordinates and the radius was the average distance of this centroid to each of the mapped points on the surface of the bamboo. For bamboo of about 2 *m* length 19 centroids were determined.

The mapping of the element's axis was obtained by projecting the centroid of each circumference on a plane normal to an imaginary axis which links the circumferential centroids of the two bamboo ends, as shown in Fig. 3. Each single mapped bamboo presented a description of the axis different from the others.

After the buckling test, the bamboo culm was cut in segments of 10 *cm* along the axis and the thickness of the bamboo's wall was taken to be the arithmetic average of 4 times the measured thickness at 90^0 distant from each other.

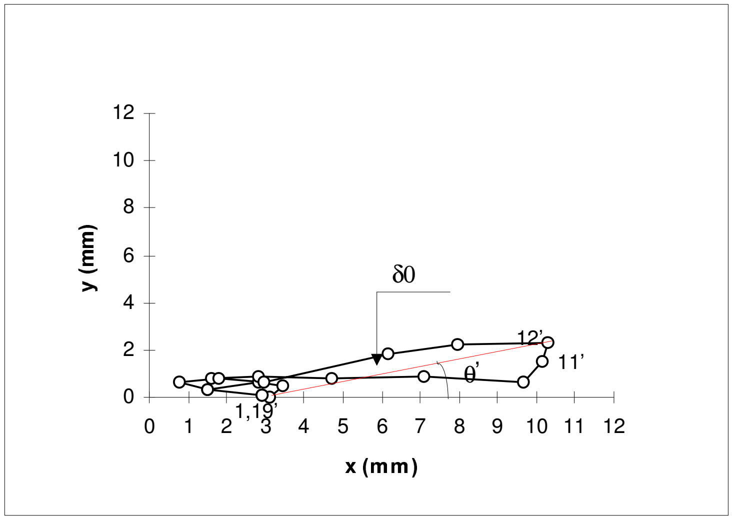

Figure 3: Projection of the Centroids

Density gradient

The fibres of the *schlerenquima* the part of bamboo's cross-section with high strength, visible as black points in Fig.1, increase their concentration when approaching the external surface. This causes the transversal section to have a density gradient in the radial direction, from the inside to outwards of the thickness, which interferes with the moment of inertia ***I*** of the section.

From observations of microstructural studies Ghavami and Marinho [4], one considers a gradient function for the radial density: - the transversal section is supposed to have constant density up to closely 75% of the thickness of the internal wall, ρ_i, and a higher density for closely to 25% of more external thickness , ρ_e .

In this way one can calculate for a transversal section, a physical inertia, I_f, larger than the geometrical inertia, or expressed differently

$$I_f = I_{gi} + I_{ge} k_1$$

where $k_1 = \frac{\rho_e}{\rho_i}$, I_{gi} is the geometrical inertia of the internal part (≈ 75 % of wall thickness t) and I_{ge} the geometrical inertia of the external part.

RESULTS OF THE GEOMETRICAL DESCRIPTION OF THE MAPPING

11 elements of about 2m length and about 10cm diameter were mapped and tested for buckling. In Fig. 3, the extreme circumferences coincide, numbers 1 ≡ 19 and represent the straight imaginary axis of bamboo. It was possible to detect the section most distant from the axis, obtaining therefore the maximum imperfection δ_0 and the orientation θ of this maximum imperfection, assuming the preferential direction of the arching of the bamboo. For this reason the LVDT – linear transducer displacement - is positioned in this direction at the center of the bamboo. The strains were also measured at the center of the element through electrical strain gauges.

Figures 4 and 5 show the geometrical inertias I_g and physical I_f, of the transversal sections, calculated for each circumferential ring along the axis of the column at a distance of 10 *cm* from each other. Analogous curves were obtained for the average radius R, the average thickness of the wall t and the average area A.

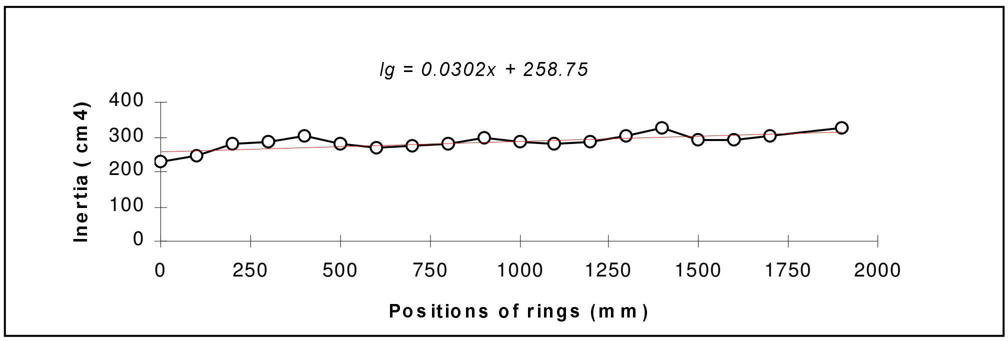

Figure 4: Geometrical inertia along longitudinal axis

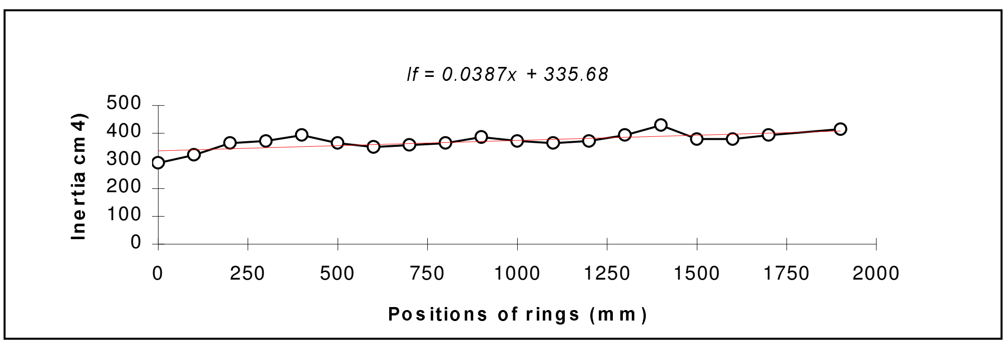

Figure 5: Physical inertia along longitudinal axis

Measurements of the gradient density showed that for bamboo with a smaller density, the density gradient causes an average of an increase of the moment of inertia for the transversal section, described by ratio I_f / I_g close to 30 %. For bamboo with a higher density this ratio falls to approximately 10%.

RESULTS OF BUCKLING TESTS

Table 1 shows the final geometrical characteristics for the studied elements, assuming as the mean value of the characteristics of the extreme transversal sections, which is approximately equal to the arithmetic average of the values obtained from the 19 transversal sections. For each element the elasticity modulus E was determined. The load Euler F_E was obtained using the Southwell Diagram and $P_{\lim}$ was obtained from the test.

Table 1: Geometry of Specimens and Test Results

Test sample	$R_e\,(mm)$	$t\ (mm)$	$I_g\ (cm^4)$	$I_f\,(cm^4)$	$F\ (kN)$	$P_{\lim}\,(kN)$
5	52,3	7,3	264	303	147,1	46,6
7	45,5	6,2	148	191	60,3	30,7
8	46,0	5,0	128	163	55,0	28,0
9	37,0	4,0	55	76	23,4	16,8
10	42,0	5,1	100	129	30,5	18,2
11	43,5	4,6	99	127	40,8	24,2
12	38,5	6,1	86	109	39,4	16,8
16	49,0	7,2	208	266	78,7	32,6
17	46,0	6,5	161	208	52,6	19,6
18	42,5	6,7	129	167	55,6	19,6
15	52,5	7,9	286	371	95,3	55,9

The instrumentation and the scheme of the buckling test are presented in Fig. 6. Fig. 7 shows the curves $P\times\delta_t$ for the tested bamboo after the total mapping, where δ_t is the total lateral displacement or the sum of the maximum imperfection δ_0 with the deflections measured at the moment of the experiment, δ .

Figure 6: Buckling tests

By plotting the values $(\delta/P)\times P$, where P is the applied load, one obtains the Southwell diagram for the bamboo, Fig. 8. The typical failure type of the element occurs by progressing squashing of the fibres in the concave side subjected to larger compression stress. The longitudinal strain gauge situated in the most compressed zone, measures the maximum longitudinal strain.

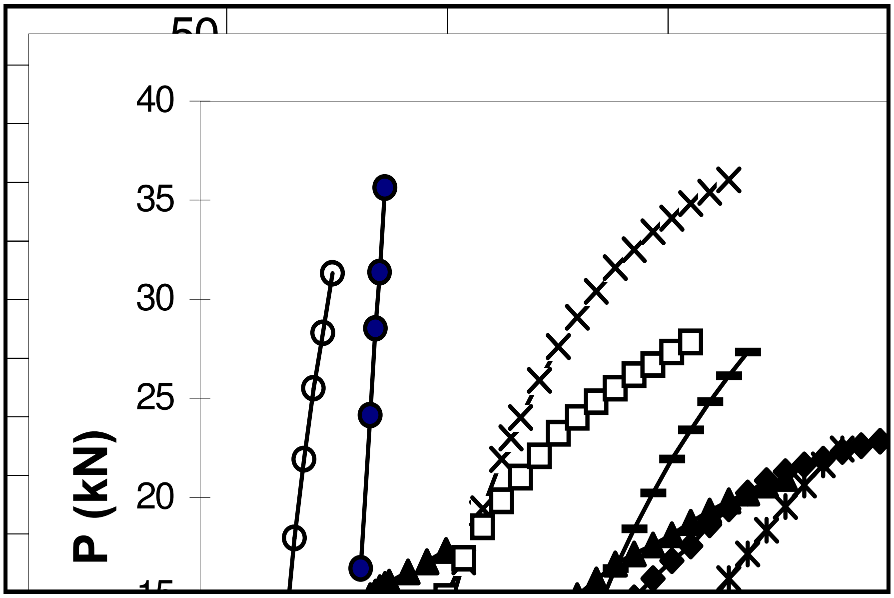

Figure 7: Curves $P \times \delta_t$

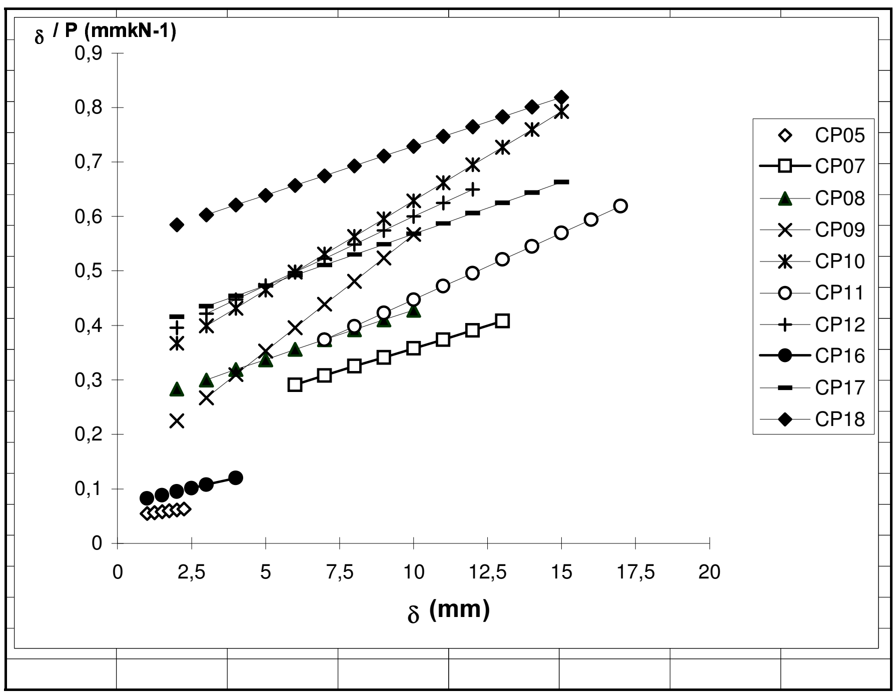

Figure 8: Southwell Diagram

ANALYSIS AND DISCUSSION OF RESULTS

Considering relatively small displacements, the lateral displacement at the centre of bamboo, is theoretically given by

$$\delta = \frac{\delta_0}{\frac{F_E}{P} - 1} \tag{1}$$

By adding δ_0 to the above expression and rearranging the terms one can write

$$\frac{F_E}{P}\delta - \delta = \delta_0 \qquad (2)$$

Therefore, when plotting $\frac{\delta}{P} \times \delta$ one obtains a straight line with an inclination of $\tan\theta = 1/F_E$ which crosses the axis of the abscissa in $\delta = \delta_0^{'}$. So, the Southwell Diagram, Fig. 8, provides the load of Euler for the experiment, as well as the initial imperfection of the element's axis. In the following it can be constructed the Table 2, which compares the imperfections obtained through the mapping δ_0, with the results $\delta_0^{'}$ of the Southwell diagram.

The difference between the values δ_0 and $\delta_0^{'}$ can be explained by the influence of some parameters of difficult control, such as:

- The presence of small excentricities in the load application.
- The fact of the Southwell Diagram being considered to be a prismatic bar and the dimensions were taken in *l/2* to the way in which bamboo has an variable inertia and the δ_0 measured in the mapping not always occurs in the center of the bamboo.

It is important to point out that the obtained results of the Southwell Diagram represent a global response of the system, or rather, by obtaining the load of Euler $F_E = \frac{1}{tg\theta}$, one obtains the rigidity to deflection $(EI)_s$ of the system, which can be expressed with

$$(EI)_s = \frac{F_E l_0^2}{\pi^2} \qquad (3)$$

As the value of E_{exp} was determined experimentally for each element, the mean value of inertia of the bamboo can be obtained by $I_s = \frac{(EI)_s}{E_{exp}}$ The comparison of these results with those inertia results obtained by the mapping and density measurements is shown in Table 2.

Table 2: Comparison of mapping results and Southwell Diagram

Cp	5	7	8	9	10	11	12	15	16	17	18
δo(mm)	7,6	9,3	16,5	7,0	11,1	8,2	14,3	7,7	3,7	19,5	30,0
δ'o(mm)	7,0	11,6	13,5	3,3	9,2	8,3	13,6	7,0	5,5	19,9	30,5
If(cm4)	303	191	163	76	129	127	109	354	266	208	167
Is(cm4)	304	170	168	65	136	120	124	373	269	196	153

It can be seen that the density gradient really affects the results, since I_s is more close to I_f than to I_g. The measuring of the maximum strain through electrical strain gauges has shown that the failure of the elements occurs mainly by progressive squashing of the fibres in the concave side of the element followed by the local buckling of the bamboo wall. Therefore,

one could assume that the maximum compressive stress responsible for the local failure of bamboo subjected to buckling would be failure stress σ_R. Nevertheless, a more profound study of the measured strain in real time, during the test, in the more compressed zone, longitudinal direction, and comparing it to the theoretical stress acting in the local, obtained through the left side of Eqn. 4, points out the limit of proportion σ_p as limit stress corresponding to limit load $P_{\lim}$. This level of stress is the transition between a stable to an unstable behavior of the bamboo wall close to the center of the element, in the concave side where the highest compression stresses occur. It is not so clear if this level of stress may also correspond to an instability of the material itself. At the moment of the local buckling phenomenon, the observed displacement of the bamboo wall was apparently simultaneous to the squashing of the fibres. It is not clear what occur first. Thus, two are the limit states to be examined for application to bamboo under axial compression: - a) the ultimate limit state, which guarantees the integrity of the column in compression, expressed by Eqn. 4.
.

$$\frac{P}{A}+\frac{P\delta_0 D}{2I_f\left(1-\frac{P}{F_E}\right)}<\sigma_p \tag{4}$$

The theoretical limit load $P_{\lim}$ will occur when the left side of the Eqn. 5 equals the right side. Under the calculated limit load, bamboo would have a theoretical lateral deflection given by

$$\delta_t=\frac{\delta_0}{1-\frac{P_{\lim}}{F_E}} \tag{5}$$

The strain limits and the maximum displacements of a structural element , known as service limit states, are mainly associated with the useful life of the materials, aesthetic factors and the comfort of the user. In the case of bamboo, where δ_0 is always present, various test specimens reached maximum loading with the bamboo quite arched, reaching tension stress in the convex side of the element, mainly in those element in which δ_0 is relatively high.

For the purpose of the project it can be considered negligible the gain of inertia I_f and to work with $I=I_g$, favorably to safety. Therefore it is possible to solve the problem in an easier way by limiting first the maximum lateral deflection. Thus let $\delta_t=n\delta_0$, and the moment of inertia of the transversal section corresponds to the element approached by $I=\pi\overline{R}^3t$. Being $F_E=\frac{\pi^2 EI}{l_0^2}$, with which Eqn.5 can be rearranged and one obtains

$$P^{\prime}_{\lim}=\frac{\pi^3 E\overline{R}^3 t(n-1)}{l_0^2 n} \tag{6}$$

We call it $P^{\prime}_{\lim}$ because the actions which load the structural element are of different combinations to verify the ultimate limit states and service limit states. When an ultimate limit state is reached, due to buckling, the structural element could fail but if the service limit state is reached there is no risk. Therefore, taken as a structural calculus, the calculus actions for verifying the ultimate limit state are always greater than those for verifying the service limit state. Therefore, if $P_{\lim}$ represents the limit load of the calculus for verifying the ultimate limit state and $P_{\lim}^{\prime}$ represents the limit load for the service limit state, one can write $P_{\lim} = \gamma \, P_{\lim}^{\prime}$, where $\gamma > 1$ is estimated for each loading. Consequently, once having calculated $P_{\lim}^{\prime}$ with Eqn. 6, Eqn. 5 is evaluated for $P_{\lim}$. It is important to say that the proposed sequence of the evaluation of these equations is a function of imperfection of the bamboo axis, which is relatively high. However, nothing prevents that the ultimate limit state is first evaluated and then the following service limit state. If the area of the transversal section is approached by $A = 2\pi \bar{R} t$, Eqn. 5 can be written as:

$$\frac{P_{\lim}}{2\pi \bar{R} t} + \frac{P_{\lim} n \delta_o}{\pi \bar{R}^2 t} < \sigma_p \tag{7}$$

CONCLUSION

Bamboo mapping was basic to the first understanding of bamboo buckling due to the number of relevant variables to the phenomenon. The adopted procedure allowed to determine the buckling plane described by $\theta^{\prime}$, the imperfections characterized by $\delta_0^{\prime}$ as well as the chaotic distribution of the centroids.

Bamboo tubes in perfect state have behaved as a Euler column with a moment of inertia I_f increased by the density gradient of the fibres in the radial direction. The consideration of the bamboo as a prismatic tube with a constant moment of inertia equal to the mean of the moment of inertia of the ends of the element provide good results for these segments whose maximum mean slenderness is 70.

The limit load of the tested elements $P_{\lim}$ were defined by the local instability of bamboo wall close to the center of the element, in the concave side of the bamboo, when the measured strain and estimated stress were both close to ε_p and σ_p, respectively, in the limit of proporcionality of the material bamboo, and not close to ε_r or σ_r which correspond to the limit of resitance of the material. In the exact moment of the failure, measured strains have increased indefinitely, resulting in the squashing of the fibres and the displacement of bamboo wall between two adjacent nodes. It was clear that the compression stress that corresponds to this phenomenon is the limit of proportionality σ_p in uniform compression but it was not clear if this level of stress may also correspond to an instability of the material bamboo itself.

The proposed equations were fitted to the test results and thus are correct to evaluate the buckling of the bamboo without splitting. It is also important to remember that these equations are working with nominal values of the loads and resistance of bamboo that is without to increase the loads or to lower the resistance by safety factors. So for the project purpose, it is necessary to add to these equations the suitable safety factors and it can be used $I = I_g$ instead of I_f, favorably to safety.

ACKNOWLEDGMENTS

The authors like to thank for the finantial support granted by FAPERJ, CNPq and CAPES. Thanks are also due to all who contributed in one way or the other in the realization of the research programs. Last but not least Ursula' s help in going through the text is much appreciated.

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